TRINITY COLLEGE

Year 12 Mathematics Applications
Test 1   2022

Section 1   Calculator Free
Data

STUDENT'S NAME ___Solutions   SP.___

DATE: Friday 25th February   TIME: 15 minutes   MARKS: 15

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (10 marks)

The ages (a) in years, and time spent playing computer games (h, in hours) per week of twelve maths teachers are shown in the table below. The equation of the least-squares line for these data is $\hat{h} = -0.4a + 62$.

Teacher	1	2	3	4	5	6	7	8	9	10	11	12
a	33	41	28	42	31	45	49	41	33	49	45	52
h	47	47	52	47	49	44	43	46	48	41	41	42

(a) Calculate the predicted h-value and the residual for teacher 11. [3]

$$\hat{h} = -0.4(45) + 62$$
$$= -18 + 62$$
$$= 44$$

residual $= h - \hat{h}$
$= 41 - 44$
$= -3$

✓ subs 45 into eqⁿ
✓ $\hat{h} = 44$
✓ residual $= -3$

The residual value for teacher 12 is 1.

(b) Plot the residuals for teachers 11 and 12 on the graph below. [2]
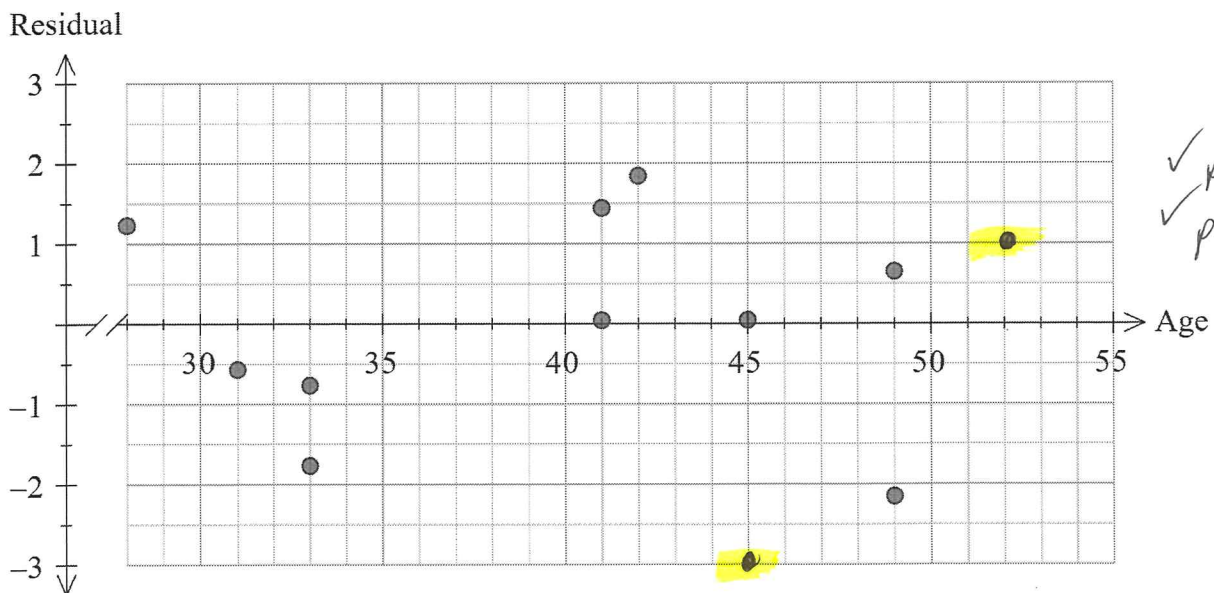
✓ plots 1 correctly
✓ plots both correctly

(c)   What do the residual values for teachers' 6 and 8 indicate?   [2]

The residual of 0 indicates that the actual data equals the predicted data. The points lie on the regression line

✓ indicates residual equals 0

✓ reasonable explanation

(d)   Justify, using the residual plot in part (b), whether the least-squares regression line is a good model for the data presented in the table.   [2]

The random nature of the residual plot suggests linear regression is a good model

✓ states yes

✓ due to random/ no pattern

The calculated correlation coefficient for these data is -0.9.

(e)   Describe how the correlation coefficient supports your response in part (d).   [1]

The strong correlation coefficient supports the use of linear regression

✓ due to strong value.

2. (5 marks)

The year level and amount of money spent per week at the canteen was recorded for 240 students and is displayed in the table below.

		Total spent in a week at the canteen			
		Less than $20	From $20 to $40	Greater than $40	Total
Year level	7 or 8	12	36	72	
	9 or 10	24	16	40	
	11 or 12	4	6	30	

(a) Identify the explanatory variable.   [1]

Year level

The percentages in each row of the following table show the proportion of each year levels spending at the canteen.

(b) Complete the table.   [2]

		Total spent in a week at the canteen			
		Less than $20	From $20 to $40	Greater than $40	Total
Year level	7 or 8	10%	30%	60%	100
	9 or 10	30%	20%	50%	100
	11 or 12	10%	15%	75%	100

(c) Using the information from the table in part (b), describe one association between these variables.   [2]

As the year group increases spending between $20 & $40 decreases eg 30%, 20%, 15%

✓ identifies an association
✓ describes in terms of differences between percentages.

TRINITY COLLEGE

Year 12 Mathematics Applications
Test 1   2022

Section 2   Calculator Assumed
Data

STUDENT'S NAME _____

DATE: Friday 25th February   **TIME:** 20 minutes   **MARKS:** 22

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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3.   (5 marks)

The maximum temperature, T, in degrees and the number of bathers sold was recorded over twelve days. The results are displayed in the table below.

Temperature, in degrees	16	18	22	24	25	28	34	34	38	39	40	42
Number of bathers sold	15	22	58	65	50	80	68	51	70	81	130	150

(a)   Determine the correlation coefficient for these data and describe the association between the variables in terms of direction and strength.   [3]

$r = 0.8164$

Strong positive relationship

✓ correct r value
✓ strong
✓ positive

(b)   The owner of the store was talking to some staff and was heard to make the comment "The warmer temperatures cause a higher number of bathers to be sold". Comment on the validity of this statement.   [2]

This statement is not valid.
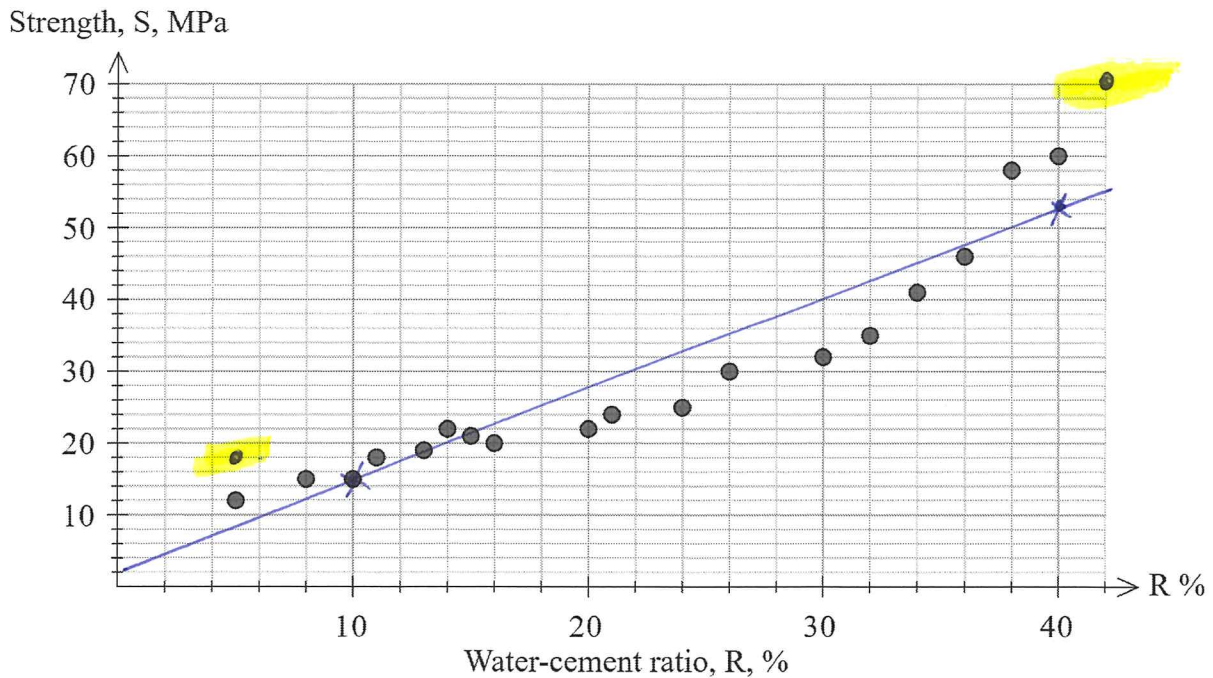This could be due to - coincidence
                      - lurking variable
                      - confounding variable

✓ states not valid
✓ gives a suitable reason

4.   (17 marks)

The following graph shows the compressive strength, in megapascals, achieved by concrete after one week for different water-cement ratios, as a percentage, used in its mixture. The study includes twenty observations.

Strength, S, MPa

(a)   Two points, (5, 18) and (42,70) have been left off the graph. Clearly add these points to the graph above.   [2]

✓ one correct
✓ both correct

Linear regression was performed on the twenty observations and the results are summarised in the table below.

Linear Regression $\hat{y} = ax + b$	
r	0.927611
a	1.274298
b	2.115443

(b)   State

    (i)   the correlation coefficient between S and R.   [1]

    $r_{SR} = 0.93$ ✓

    (ii)   the equation of the least-squares regression line that can be used to predict S from R.   [2]

    $\hat{S} = 1.2743R + 2.1154$

    ✓ correct equation
    ✓ in terms of S & R.

(c)   Add the least-squares line to the scatterplot, clearly indicating the points used.   [2]

✓ correct line
✓ clearly shows two points.

$\hat{S}(10) = 14.86$
$\hat{S}(40) = 53.01$

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(d)   In the context of the question, interpret the slope of the least-squares line.   [2]

For every 1% increase in water-cement ratio the strength increases by an average of 1.27 mPa

(e)   What percentage of the variation in strength can be explained by the variation in ratio?   [1]

$r^2 = 0.86$

∴ 86% ✓

(f)   Predict the strength of cement for a water-cement ratio of 80% and comment on the reliability of the prediction.   [3]
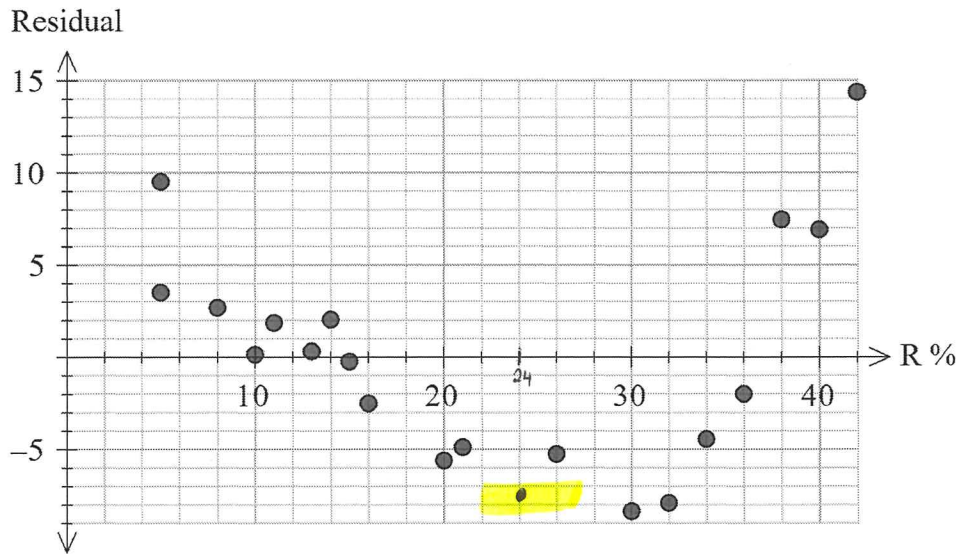
$\hat{S}(80) = 1.2743(80) + 2.1154$

$= 104.059$

$\approx 104 \; mPA$

Despite strong r value we are extrapolating ∴ not likely to be reliable

✓ ≤ 104 mPA
✓ not reliable
✓ due to extrap.

The following graph shows the resulting residual plot after performing linear regression.

(g)   Plot the residual for the point (24, 25) on the graph above.   [2]

$\hat{S} = 32.6986...$

$S - \hat{S} = 25 - 32.7$

$= -7.7$

(24, -7.7)

✓ residual = -7.7
✓ plots their residual

(h)   Justify using the residual plot, whether the least-squares regression line is a good model for these data.   [2]

Given the pattern in the residuals, linear regression is not a good model for these data

✓ states there is a pattern
✓ concludes not good model